SEE. VOL 14 No. 1

THE APPLICATION OF A VECTOR AUTOREGRESSIVE MODEL TO MONEY, INCOME AND PRICE LINKS IN THE SOUTH AFRICAN ECONOMY

G D I Barr and B S Kantor*

Abstract

Vector autoregression (VAR) is a frequently used technique for analysing the dynamic impact of random shocks on systems of interrelated variables. It is particularly applicable to analysing the interrelationship between simultaneously determined endogenous variables which exhibit bicausal behaviour due to the existence of feedback.

In small open economies the simultaneous determination of equilibrium prices, money and exchange rate lends itself to such an analysis. This paper considers the application of VAR to South African exchange rates, prices and money over the past 10 years using monthly data ana analyses the impulse response function of each variable to the other. For these variables VAR yields far more plausible results than conventional methods.

1. Introduction

The task of separating the cause and effect of changes in the supply of money on income or prices is not a simple one. Complex feedback mechanisms whereby changes in income and prices influence the demand for money and in turn the supply of money, depending on the policy reactions of the monetary authorities, complicate the task of precisely identifying the impact of any change in the supply of money.

The purpose of this paper is to separate money supply causes and effects in South Africa by applying a vector autoregressive model (a class of linear stochastic difference equations) to the relevant South African economic time series. This class of models has the great advantage of allowing the parameter estimates to be interpreted in a theoretical context and may also be used to predict how the equilibrium values of the important variables would change in response to hypothetical changes or 'shocks' in the environment of the model. Modelling of business cycles using stochastic difference equation systems can be traced back to Frisch (1933). More recently work by Sims (1972, 1980), Hansen and Sargent (1980, 1981) and Sargent and Wallace (1985) have provided an important challenge to the conventional econometric models of macroeconomic systems.

*Respectively associate professor, Departments of Mathematical Statistics and Economics and professor, Department of Economics, University of Cape Town, Private Bag, Rondebosch 7700, Republic of South Africa.

2. Economic tests of the links between money and income: A brief history

It was the work of Milton Friedman and Anna Schwartz (Friedman and Schwartz, 1963) on the monetary history of the US that did most to revive interest in the statistical relationship between money and income. These links were examined further by Leonall Anderson and Jerry Jordan in the form of what became the famous St Louis equations (Anderson and Jordan, 1968). Such equations are one sided distributed lag models using changes in nominal income as the dependent variables and changes in money supply and perhaps one other variable (e.g. a measure of fiscal policy) as the independent variables. The St Louis system has the general form

$$y_t = \sum_{1=0}^{\infty} \beta_i m_{t-1} + \sum_{1=0}^{\infty} \gamma_i g_{t-i} + e_t.$$

The original St Louis equation specified monetary aggregates and high employment federal government expenditures for the US as the exogenous variables[1]. The coefficients of the lagged variables were constrained to be on a fourth degree polynomial with end point coefficients for both variables constrained to equal zero.

The estimates of the coefficients indicated that the impact of changes in money supply was statistically significant, while that of fiscal policy was barely so (Anderson and Jordan, 1968). These results generated a highly critical response from Keynesian economists. In particular it was argued that while St Louis equations may be regarded as the reduced form of a larger model, the equations were not derived explicitly from a larger model and therefore important other exogenous variables may be omitted. If so the β_i could be regarded as 'mongrel' coefficients not indicative of the average response of y to exogenous impulses in m (see Modigliani and Ando (1976), Sargent (1987)).

Furthermore and perhaps most importantly, if the right hand side variables are in fact not exogenous with respect to income, or in other words, income influences the money supply and the money supply also influences income, the least squares estimates of the coefficients would reveal simultaneous equation bias.

The work of Sims (1972) on causality went some way to resolving this debate. Sims' notion of unidirectional causality yielded the following easily testable result. "If and only if causality runs one way from current and past values of some list of exogenous variables to a given endogenous variable, then in a regression of the endogenous variable on past, current and future values of the exogenous variables the future values of the exogenous variables should have zero coefficients" (Sims, 1972: 541).

Sims applied this test to the two-variable system of Andersen and Jordan and tested the hypothesis that causation runs one way from income to money. Although he found

[1]In our own work we have applied such polynomial distributed lag procedures to South African data (see Barr and Kantor (1982, 1986)).

the data was consistent with a null hypothesis that causality runs entirely from money to income without feedback there still exists disagreement about how far Sims work goes in confirming the St Louis approach as an accurate way of estimating money multipliers. Nevertheless Sims had provided a structure within which one could at least in principle, subject the Keynesian claims against the St Louis approach to statistical testing.

The Sims (1972) test along with Granger's (1969) test of causality was applied to quarterly South African money and GDP data over the period 1965 to 1978 by Barr and Dietzsch (1980). These tests were consistent in rejecting the hypothesis of unidirectional causality from money to income or vice versa. It was concluded at the time that the reduced form methods were perhaps not sophisticated enough to expose the existence of some feedback in the money-income relationship. Since both money and income were at least to some extent endogenous in the South African context, it was felt that these causality issues had to be considered within the framework of a more comprehensive model in which money and income were endogenized.

The extension of single equation St Louis approaches to models with a number of interlinked endogenous variables, where feedback mechanisms exist, has become a challenging area for further econometric research. In a later article, Sims (1980) criticised the use of large econometric models for the analysis of systems which exhibit bidirectional causality or feedback. Sims argued that in reality the coefficients of these large models are almost always unidentified because the cross equation restrictions are often not operative in practice. He cited work by T C Lui (1960) and Hatanaka (1973) which pointed to implicit, often unreasonable, a priori assumptions regarding the serial correlation structure of the errors when identification is tested. Sims went on to show how the incorporation of rational expectations often undermines many of the exclusion restrictions econometricians consider as being reliable. These criticisms led Sims to investigate the possibility of building multi equation models with a minimum of restrictions. Sims argued that any restrictions which are introduced should be applied more systematically which then "could lead to the capture of empirical regularities which remain hidden to the standard procedures and hence lead to improved forecasts and policy prorejections" (Sims (1980, p.14)). In particular he questions the approach of parsimonious estimation and states that, in contrast, profligately parameterised macro-models may have advantages.

Sims suggests that the class of unconstrained multivariate time series models are appropriate for this task and in particular that a vector autoregression be used as a basic model.

The vector autoregressive system expresses each of a set of endogenous variables as a function of lagged values of itself and lagged values of all the other endogenous variables in the system. It thus allays Sims' concern regarding the restrictions that are placed on this system as it is (apart from the lag specification) completely unrestricted. Because of the large number of variables in each equation, however, the number of endogenous variables that can be considered remains rather small.

3. Application of the vector autoregressive approach to the South African case

The data for this study was collected on a quarterly basis over the period 1966 (1st quarter) to 1988 (final quarter). Apart from the income, money and price variables the balance of payments was incorporated because of the open character of the South African economy. The series considered were:

(i) GDP at constant 1985 prices (GDP).

(ii) Rand value of notes in circulation (M3 vas considered as an alternative) (M).

(iii) Private Consumption Expenditure Deflator (P).

(iv) The ratio of the trade balance to nominal GDP (BOP).

The first three series were considered in log year-on-year growth rate form to remove the time trend and its associated strong autoregressive component as well as any seasonality in the data. GDP, P and BOP which are rather volatile were then smoothed using a central 5th order moving average (see Figures 1-4). (This procedure does not materially affect any of the results but does smooth out the impulse-response functions and makes them easier to interpret.)

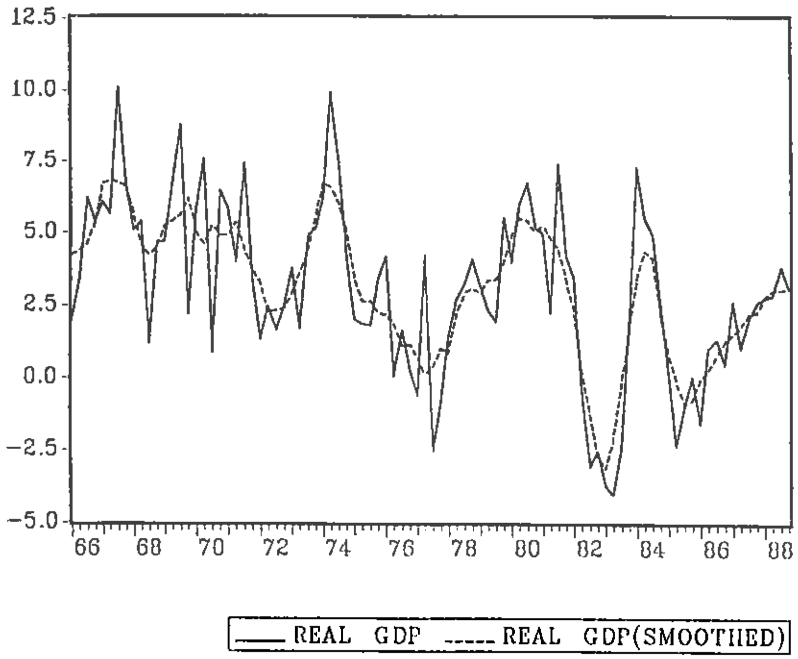

Figure 1: Real GDP at constant 85 prices

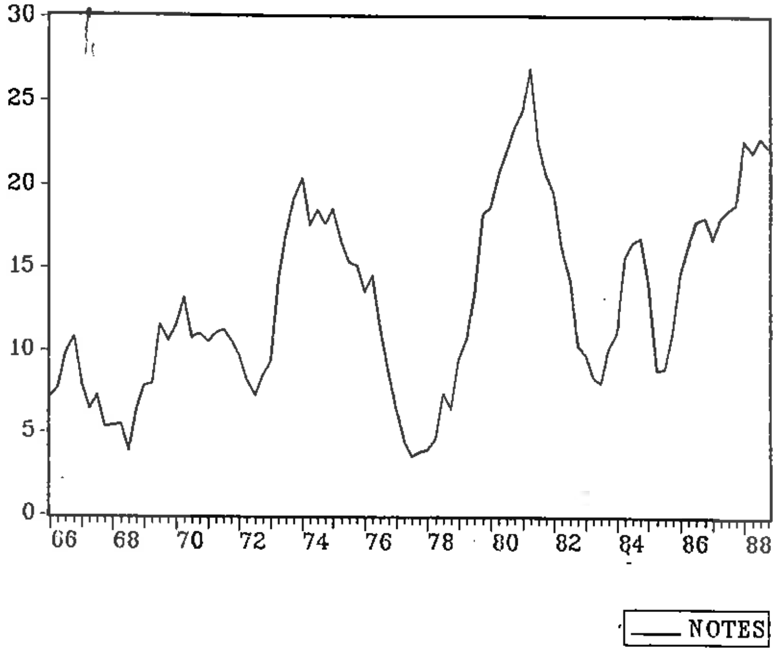

Figure 2: Notes in circulation

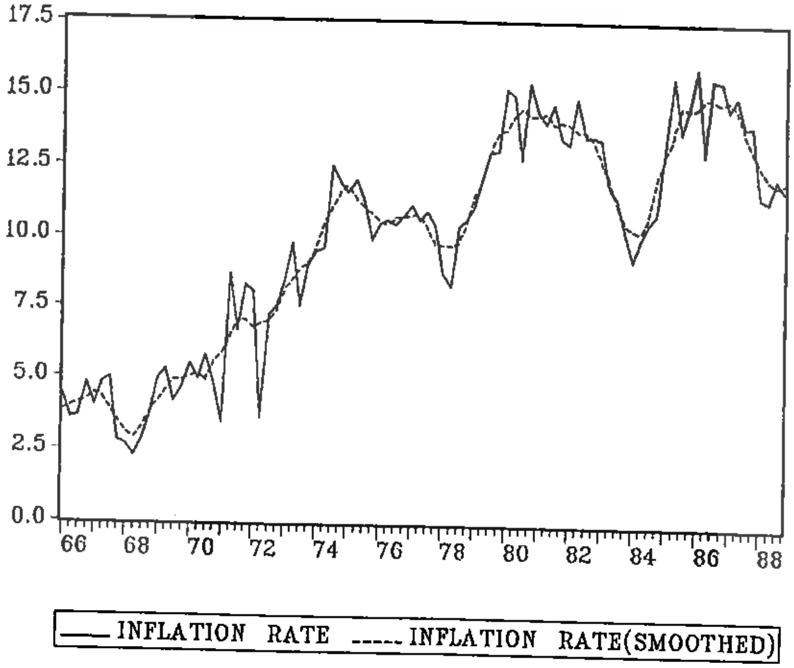

Figure 3: Inflation rate

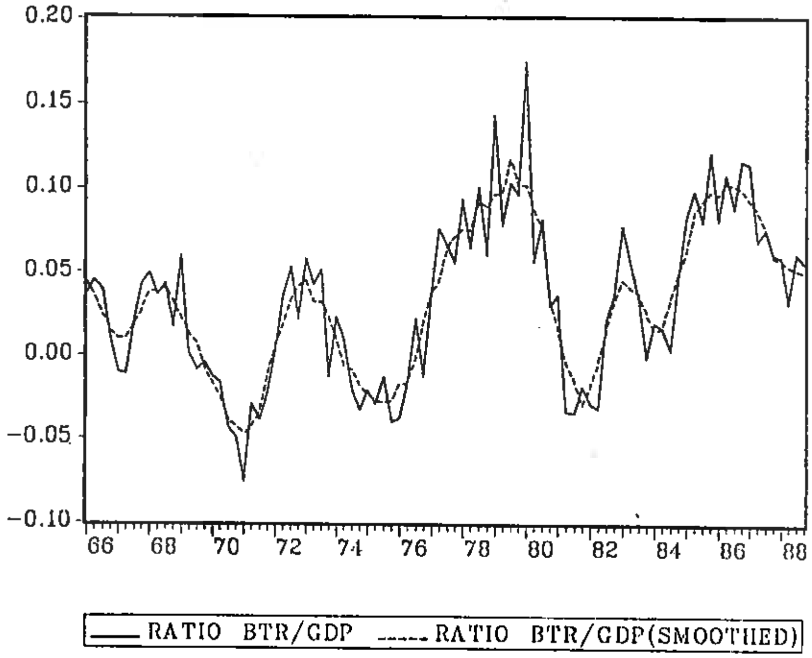

Figure 4: Ratio of trade balance to GDP

The system was estimated as a general vector autoregression. Economic theory would tend to dictate that the maximum number of lags used in such a specification would be about eight. A shorter lag length of four was tested (against eight) using Sims (1980) adapted likelihood ratio test viz:

$$(T - K) \quad \text{Log} \quad \frac{|\, D_R \,|}{|\, D_U \,|}$$

where T is the sample size,

K the total number of regression coefficients divided by the number of equations,

D_R matrix of cross products of residuals when the model is restricted,

D_U above for the restricted model.

The specification of lag length eight was treated as the unrestricted form and the specification with lag four as the restricted form. The shorter lag length was not significantly different from the longer lag length with $\chi^2(64) = 67.09$ with a corresponding significance of about 0.35 and was used in the subsequent analysis.

The unconstrained vector autoregressive system would thus have the following form:

$$1. \quad GDP_t = \sum_{i=1}^{4} \beta_{1Gi} GDP_{t-i} + \sum_{i=1}^{4} \beta_{1Mi} M_{t-i} = \sum_{i=1}^{4} \beta_{1Pi} P_{t-i} + \sum_{i=1}^{4} \beta_{1Bi} BOP_{t-i} + e_{1t}$$

$$2. \quad M_t = \sum_{i=1}^{4} \beta_{2Mi} M_{t-i} + \sum_{i=1}^{4} \beta_{2Gi} GDP_{t-1} + \sum_{i=1}^{4} \beta_{2Pi} P_{t-1} + \sum_{i=1}^{4} \beta_{2Bi} BOP_{t-i} + e_{2t}$$

$$3. \quad P_t = \sum_{i=1}^{4} \beta_{3Pi} P_{t-i} + \sum_{i=1}^{4} \beta_{3Gi} GDP_{t-i} + \sum_{i=1}^{4} \beta_{3Mi} M_{t-i} + \sum_{i=1}^{4} \beta_{3Bi} BOP_{t-i} + e_{3t}$$

$$4. \quad BOP_t = \sum_{i=1}^{4} \beta_{4Bi} BOP_{t-i} + \sum_{i=1}^{4} \beta_{4Gi} GDP_{t-i} + \sum_{i=1}^{4} \beta_{4Mi} M_{t-i} + \sum_{1=1}^{4} \beta_{4Pi} P_{t-i} + e_{4t}$$

The individual estimates of the ß's are not easy to interpret because of the complicated cross-equation feedback mechanisms that exist in such a system. Summary statistical results are as follows for the 4 equations.

Dependent variable	R^2	F	D.W.[2]
GDP	0.957	122.4	1.965
M	0.911	56.77	1.949
P	0.992	680.34	1.906
BOP	0.961	138.07	2.031

4. System simulation when shocks are introduced

Sims (1980) recommends that the best way to interpret such a system is via its response to typical random shocks on each equation.

The shocks are positive residuals of one standard deviation (of the residual) in magnitude for each equation. Thus the money shock on GDP represents the effect of a one standard deviation increase in e_{2t} on GDP over subsequent quarters. The money shock impacts directly onto money in the first place and then GDP after one lag. Money also influences GDP via its impact on prices and the balance of payments (through the correlation that exists between e_{2t} and e_{3t} and e_{4t}) and then subsequently back to GDP. The set of direct, indirect and feedback responses captured in such an analysis are most easily considered graphically.

It should be noted that generally the estimated residual vectors in the four equations will be contemporaneously correlated. In order to view the distinct effects of a shock

[2] D.Ws are biased because of the lagged dependent variable.

on any equation it would be required that the shocks are orthogonal. There is, however, no unique best way to orthogonalise these shocks. For example, in the case of e_{1t} and e_{2t} an orthogonalising transformation could attribute the correlated component to either e_{1t} or e_{2t}. The method most commonly used to orthogonalise the errors is to triangularise the vector of residuals in the order in which the variables are presented. This imposes a unidirectional causal chain on the residuals. Thus a shock to equation one impacts contemporaneously on to itself, equations two, three and four; a shock to equation two impacts contemporaneously onto itself, equations three and four (but not equation one); a shock to equation three onto equation four (but not equation one or two); a shock to equation four onto itself alone.

4.1 The shock response functions for each variable

Figures 5-7 below capture for each set of orthogonal shocks in the triangularised system, the response of each of the four variables in our system. Thus for example in Figure 5 we consider the effect of orthogonal shocks to GDP, M, P and BOP on GDP.

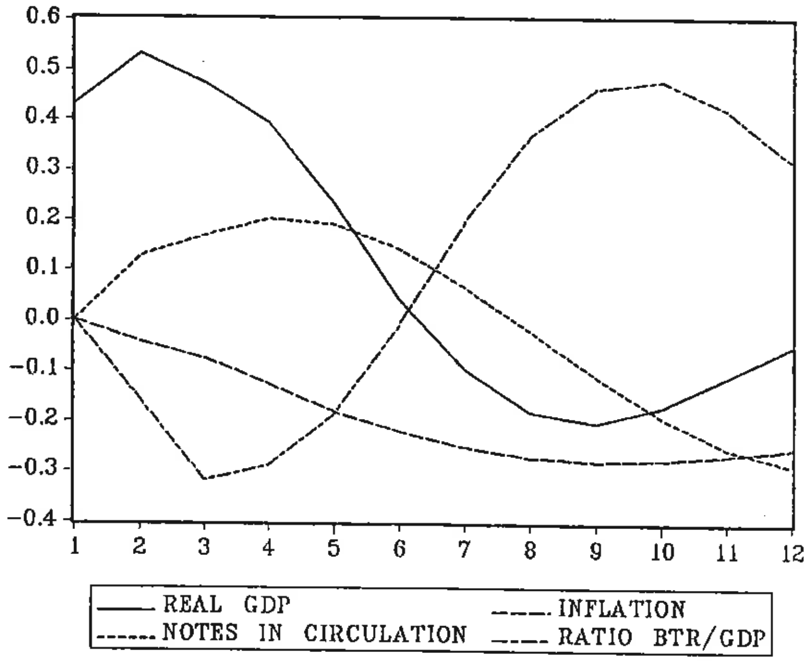

Figure 5: Responses of REAL GDP to One Std Deviation Shocks

In Figure 5 the impact on real GDP of respective shocks to real GDP itself, the money supply, the inflation rate and the balance of payments is illustrated. As may be seen, real GDP will respond positively to a money supply shock, with the real income response tailing off after 8 quarters. A shock to the inflation rate is shown to lead to a persistent decline in real GDP, while a balance of payments shock causes GDP growth to decline sharply followed by a later recovery. It should be recognised that a shock to the trade balance is often associated with a negative shock to the capital account of the balance of payments. The result shown in Figure 5 that a shock in the form of capital withdrawals would lead, at first to a sharp decline in and then to a recovery in real GDP, as the trade balance effects work through on income, is perhaps a convincing one.

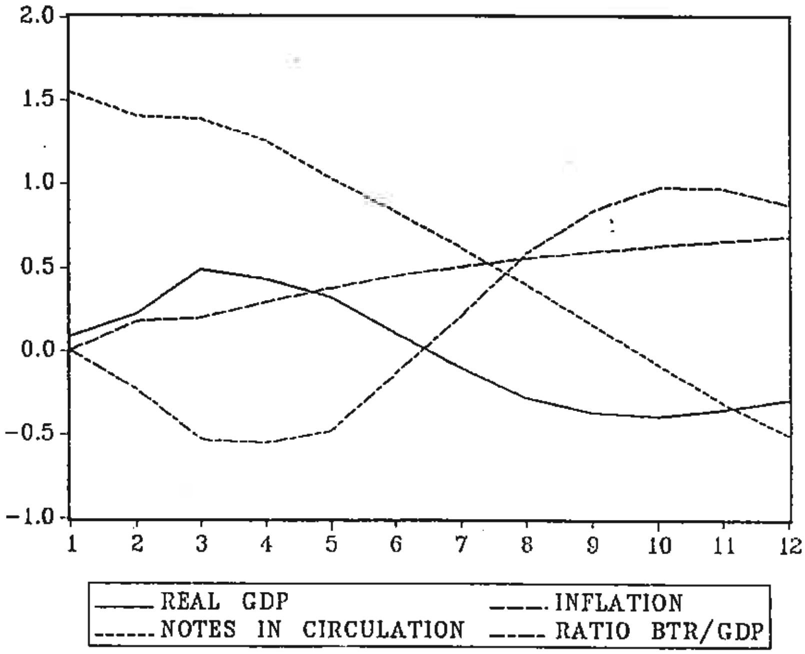

Figure 6: Response of GNOTES to One Std Deviation Shocks

The impact of a shock on GDP and inflation on the growth in the money supply is illustrated in Figure 6. As may be seen the impact of a shock increase in real GDP on money is not very powerful and peaks after about 6 quarters. Thus it would appear that the impact of a shock to the money supply on GDP is of much greater practical relevance than the reverse effect of a GDP shock on money supply. To emphasise this point it may be useful to compare actual magnitudes rather than the time path of the response functions. A one standard deviation shock to real GDP growth is of the order of 0,45% per annum. A one standard deviation shock to money growth is about 1½% p.a.[3]

[3] Average growth rates in real GDP have been 3,01% while average growth in the note issue averaged 13,3% over the period. The M3 growth rate averaged 13,67% over the period 1967 (1st quarter) to 1988 (4th quarter).

Thus a one standard deviation shock in real GDP of 0,45% would add a maximum of an extra $^1/2$% increase in money supply growth rates realised after 3 quarters. A one standard deviation increase in the note issue of $1^1/_2$% would cause real GDP growth rates to increase gradually by as much as 0,2% realised after 5 or 6 quarters.

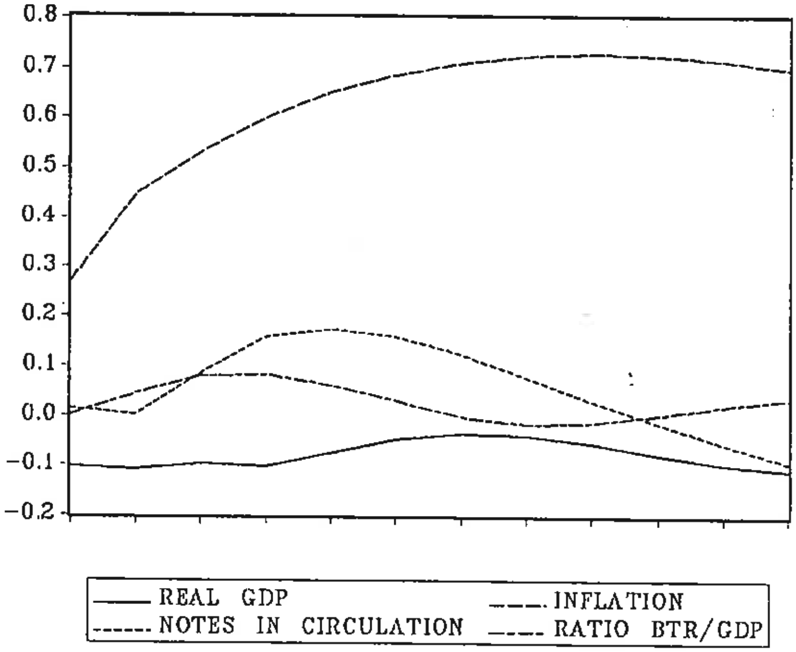

Figure 7: Response of INFLATION to One Std Deviation Shocks

In Figure 7 the impact of the respective shocks on the inflation rate is illustrated. A monetary shock will cause inflation to increase as will a balance of payments shock. A positive shock to real GDP will cause inflation to decline. As may be seen in Figure 6 an inflation shock will also cause money supply to grow more rapidly. The order of magnitude of these causes and effects of inflation are as follows. A one standard deviation shock to inflation is of the order of 0,3%. Therefore as may be seen in Figure 6, a 0.3% shock increase in the inflation rate will gradually increase pressure on money supply growth, by as much as 0,5% after 3 years. According to Figure 7, a 1,5% increase in money supply growth will increase the inflation rate by 0,15% within 6 quarters.

5. Conclusion

The analysis has confirmed the independent importance of money supply growth for the growth in real incomes and inflation in South Africa. The study also shows that influences running from income or prices to money are also present. In other words the evidence is that money supply growth in South Africa can be regarded, for the purposes of the analysis of money, as partly exogenous and partly endogenous.

References

Anderson, LC and Jordan, JL (1968): "Monetary and Fiscal Actions: A Test of their Relative Importance in Economic Stabilization", *Federal Reserve Bank of St. Louis Review*, 50(11), 11-23.

Barr, GDI and Dietzsch, CH (1980): "The Direction of Causality between Money and Income: The South African Case", *Journal for Studies in Economics and Econometrics*, 7, 24-47.

Barr, GDI and Kantor, BS (1982): "Money and Economic Activity", *South African Journal of Economics*, 50(4).

Friedman, M and Schwartz, AJ (1963): *A Monetary History of the United States 1867-1960*, Princeton University Press.

Frisch, R. (1933): "Propagation Problems and Impulse Problems in Dynamic Economics", in *Economic Essays in Honor of Gustav Cassel*, London: Allen and Unwin.

Hansen, LP and Sargent, TJ (1980): "Formulating and Estimating Dynamic Linear Rational Expectations Models", *Journal of Economic Dynamics and Control*, 2 (1), 7-46.

Hansen, LP and Sargent, TJ (1981): "Linear Expectations Models for Dynamically Interrelated Variables", in Lucas, RE (Jr) and Sargent, TJ (eds.); *Rational Expectations and Econometric Practice*, Minneapolis.

Hatanaka, M (1975): "On the Global Identification of the Dynamic Simultaneous Equation Model with Stationary Disturbances", *International Economic Review*, 16, 545-54.

Kantor, BS and Barr, GDI (1986): "The Impact of a Change in the Price of Petrol on the South African Rate of Inflation, *Journal for Studies in Economics and Econometrics*, 26, 35-58.

Lui, TC (1960): "Underidentification, Structural Estimation and Forecasting", *Econometrics*, 28, 855-861.

Modigliani, F and Ando, A (1976): "Impacts of Fiscal Actions on Aggregate Income and the Monetarism Controversy: Theory and Evidence", in Stein, JL (ed.), *Monetarism, 1, Studies in Monetary Economics*, North-Holland, 17-42.

Sargent, TJ and Wallace, N (1985): "Identification and Estimation of a Model of Hyperinflation with a Continuum of 'Sunspot' Equilibria", *Discussion paper No. 220, Centre for Economic Research*, University of Minnesota.

Sargent, TJ (1987): *Macroeconomic Theory*, 2nd edition, Academic Press.

Sims, CA (1980): "Macroeconomics and Reality", *Econometrica*, 48(1), 1-48.

Sims, CA (1972): "Money, Income and Causality", *American Economic Review*, 62(4), 540-552.